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Title: General theory of sub-quarterwave multilayers with highly absorbing materials

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Abstract

A general theory of multilayers with enhanced reflectance has been developed based on the superposition of sub-quarterwave layers of various highly radiation absorbing materials. The theory has been developed by second order expansion of the multilayer reflectance with respect to the optical constant differences between the materials in the multilayer. The current paper completes and improves the theory that was developed in a previous article[*] by including the case of non-normal incidence and general radiation polarization, and by providing more accurate film thickness values of the optimized multilayer than with the previous theory. The theory provides an accurate approach to the design of a new concept of multilayer coatings with more than two materials. The new multilayers are particularly adequate to enhance the reflectance of the materials in the far and extreme ultraviolet.

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1. Introduction

Efficient multilayer coatings with high reflectance over the extreme ultraviolet (EUV) spectral range ~50-125 nm have been precluded so far because all the available materials strongly absorb radiation. A new theory has been recently derived[1] on multilayers with sub-quarterwave films of highly radiation absorbing materials, where reflectance enhancements over the reflectance of single layers were obtained. The theory was developed by the expansion of the multilayer reflectance up to the first order in the optical constant differences between the materials in the multilayer. Even though the obtained mathematical derivations were valid in any spectral range, they were particularly adequate for the EUV, where transparent materials for standard multilayers are not available. According to this theory, reflectance enhancements can be obtained by the superposition of thin films of various materials with certain material selection rules. The theory was developed for radiation impinging on the multilayer at normal incidence. Simple equations were determined for the optimum film thicknesses, which provided the right order of magnitude.

In the present paper the sub-quarterwave multilayer theory of Ref. 1 is redeveloped. The theory is generalized to design multilayers optimized to provide the highest possible reflectance at any angle of incidence and any radiation polarization. With the current theory the reflectance is expanded up to the second order in the optical constant differences between the materials. This gives rise to new equations providing more accurate values of the optimum film thickness. The current multilayers can find applications mainly in the spectral region ~50-125 nm, although the equations presented here are applicable all over the spectrum. Outside the above interval high normal reflectance is regularly obtained by use of highly reflective materials (above 115 nm) or multilayer coatings (both below ~50 nm and above ~125 nm). In Section 2.1 we redevelop the first order expansion of the reflectance, resulting in a more accurate theory than in Ref. 1, and in a more general theory, addressing any incidence angle and polarization. Section 2.2 analyzes the way to select the materials for the sub-quarterwave multilayers and the expected reflectance enhancements. The second order expansion of the reflectance is included in Section 2.3, which provides more accurate film thickness values for the highest possible reflectance. Section 3 is devoted to two examples of the current theory. In subsection 3.1 we compare the accuracy of the current theory to that of Ref. 1. In Subsection 3.2 we show an example of a multilayer optimized for non-normal incident radiation.

2. Theory of sub-quarterwave multilayer coatings with highly absorbing materials

We will assume a multilayer coating consisting of m thin films of different absorbing materials deposited onto an opaque substrate. The refractive indices of the films and substrate are referred to as N_i, i=1, m+1 starting with the outermost film. All the N_i are complex numbers. In the more general case all N_i may be different. Reflectance of the multilayer is obtained through recurrent calculation from the innermost to the outermost interfaces using the following equation:

$$g_{i-1} = \frac{f_{i-1} + g_i \exp(\beta_i)}{1 + f_{i-1} g_i \exp(\beta_i)} \qquad (1)$$

with:

$$g_m = f_m \qquad (2)$$

f_i stands for the Fresnel coefficient at the *i*- *i*+1 interface, g_i stands for the amplitude reflectance at the same interface including all the contributions from the inner films, and β_i represents a function of the radiation propagation through the *i*-th film:

$$\beta_i = \frac{4\pi i N_i \cos\theta_i x_i}{\lambda} \qquad (3)$$

where *i* is the imaginary unit, λ is the wavelength, x_i is the *i*-th film thickness and θ_i is the complex propagation angle at the *i*-th film given by:

$$N_i \cos\theta_i = \sqrt{N_i^{\,2} - N_0^{\,2} sin^2\theta_0} \qquad (4)$$

θ_0 stands for the angle of incidence of the radiation from a non-absorbing incidence medium with the refractive index N_0. In order to make the theory more general, radiation is assumed to impinge on the multilayer from a medium placed between the outer medium and the multilayer. This medium with refractive index N_{Inc} may be absorbing in general. N_0 and θ_0 will be used only to define the Snell invariant N_0 sinθ_0. It is satisfied:

$$N_{Inc} sin\theta_{Inc} = N_0 sin\theta_0 \qquad (5)$$

By defining N_{Inc} as an absorbing medium the expressions obtained below are also valid when we extract a sub-multilayer starting with the *i*-th film down to the opaque substrate; in that case the medium where radiation impinges on the sub-multilayer is: $N_{Inc} = N_{i-1}$. When radiation impinges from the outer medium N_0 onto the outermost film of the multilayer, then: $N_{Inc} = N_0$. A second advantage of proceeding in that manner is that the multilayer may have an outermost layer, such as a protective layer, which might not be optimized in general according to the current theory.

The amplitude reflectance of the multilayer is given by:

$$r = g_0 = \frac{f_0 + g_1 \exp(\beta_1)}{1 + f_0 g_1 \exp(\beta_1)} \qquad (6)$$

where f_0 is the Fresnel coefficient at the interface with refractive indices N_{Inc} and N_1.

The purpose of this theory is to obtain a mathematical relationship between N_{Inc}, and N_i, i=1 to m+1, in order to derive material selection rules to construct multilayers providing a reflectance enhancement through every layer. In order to obtain the general trends of the reflectance as a function of N_{Inc}, and N_i, i=1 t o m+1, Eqs. (1), (2) and (6) are difficult to handle. Therefore, we will limit calculations to the particular case where all the materials in the multilayer have close refractive indices. We define $\Delta N_i = N_{i+1} - N_i$, i=1 to m, with $\Delta N_i = \Delta n_i + i\Delta k_i$. The above assumption implies $|\Delta N_i| << |N_i|$, and hence $|\Delta n_i|, |\Delta k_i| << |N_i|$. As it was demonstrated in Ref.1, the applications of sub-quarterwave multilayers are not restricted to materials with close refractive indices; however, the above approximations are a simple method to derive the theory.

The theory will be developed as follows. First, the multilayer reflectance will be expanded to the second order in ΔN_i. In Section 2.1 the first order expansion will be used to obtain the material selection rules and the initial film thickness values. In a second step, more accurate thickness values will be obtained in Section 2.3 using the expansion up to the second order in ΔN_i, starting with the thickness values obtained from the first order expansion as the initial values.

Coefficients f_i and g_i, i=1 to m, are first order functions of ΔN_i. The term $f_{i-1}g_i$ in the denominator of Eq. (1) is of second order in ΔN_i for $i \geq 2$. Since the numerator of Eq. (1) is of first order in ΔN_i, the term $f_{i-1}g_i \exp(\beta_i)$ gives a third order contribution to g_{i-1} and will be neglected:

$$g_{i-1} \approx f_{i-1} + g_i \exp(\beta_i) + O(\Delta N^3), \quad i = 2 \quad to \quad m \tag{7}$$

We obtain g_1 recurrently:

$$g_1 = \sum_{i=1}^{m} f_i \exp\left(\sum_{j=2}^{i} \beta_j \right) + O(\Delta N^3) \tag{8}$$

In Eq. (6) we cannot neglect the term $f_0 g_1$ in the denominator because f_0 is a zero order function of ΔN_i. Expansion of Eq. (6) up to the second order in ΔN_i yields:

$$r = r_{Inc,1} + \left(1 - r_{Inc,1}{}^2\right) \; \bar{t}_1 - r_{Inc,1}\left(1 - r_{Inc,1}{}^2\right) \; (\bar{t}_1)^2 + O(\Delta N^3) \tag{9}$$

where

$$\bar{t}_i = \sum_{j=i}^{m} t_j, \qquad t_j = f_j \exp\left(\bar{\beta}_j\right), \qquad \bar{\beta}_j = \sum_{k=1}^{j} \beta_k \tag{10}$$

$r_{Inc,1}$ stands for the amplitude reflectance given by the Fresnel coefficient at the interface with refractive indices N_{Inc} and N_1. The second order expansion of the reflectance is:

$$R = R_{Inc,1} + 2\,\mathrm{Re}\left[s\bar{t}_1\right] - 2\,\mathrm{Re}\left[r_{Inc,1} s\left(\bar{t}_1\right)^2\right] + \left|1 - r_{Inc,1}{}^2\right|^2 \bar{t}_1{}^* \bar{t}_1 + O\left(\Delta N^3\right) \tag{11}$$
$$s \equiv r_{Inc,1}{}^* \left(1 - r_{Inc,1}{}^2\right)$$

Re[ ] stands for the real part of the expression in brackets. The asterisk refers to the complex conjugate. In Eq. (11) the terms in $R_{Inc,1}$, $\bar{t}_1$, and $\left(\bar{t}_1\right)^2$ and $\bar{t}_1{}^*\bar{t}_1$, are the zero, first and second order contributions to reflectance, respectively. The zero order contribution is the reflectance of the outer interface, as if the outermost film were opaque. The first order term contains the contributions of the radiation that was reflected at only one internal interface and left the multilayer with no further reflection. The second order term contains the contributions of the radiation that was reflected at one internal interface, then reflected back at the outermost interface, later reflected in one of the internal interfaces, and finally left the multilayer with no further reflection. Thus, the assumption of small refractive index differences results in a small reflectance at every internal interface, which allows neglecting the radiation reflected at three or more internal interfaces.

In Eqs. (9), (10) and (11) the exact Fresnel coefficients (f_j) were used instead of their expansions because the exact values are easily calculated. There are several reasons to proceed in this manner: the derived equations are simpler, more accurate, and they are valid for *s* and *p* polarization, depending on the Fresnel coefficients used. Nevertheless, the Fresnel coefficients will be expanded in a few cases below where this gives rise to more informative equations. Contrarily, in the theory developed in Ref. 1 the Fresnel coefficients were expanded and all the refractive indices were referred to the refractive index of the outermost film.

We now search for the *m*-dimensional reflectance maxima in the thickness space. The optimum film thickness $\{x_{max,l}\}$ =($x_{max,1}$, $x_{max,2}$,...,$x_{max,m}$) of the multilayer will make zero the first derivative of the reflectance with respect to every film thickness:

$$\frac{\partial R}{\partial x_i} = 2\,\mathrm{Re}\left[s\frac{\partial \bar{t}_1}{\partial x_i} \right] - 4\,\mathrm{Re}\left[r_{Inc,1} s\bar{t}_1 \frac{\partial \bar{t}_1}{\partial x_i} \right] + 2\left|1 - r_{Inc,1}{}^2\right|^2 \mathrm{Re}\left[\bar{t}_1{}^* \frac{\partial \bar{t}_1}{\partial x_i} \right] = 0, \quad i = 1 \;\; to \;\; m \quad (12)$$

The second derivative must satisfy the following conditions:

$$(-1)^{m-i} \det_{i,m}\left(\frac{\partial^2 R}{\partial x_j \partial x_k} \right) < 0, \quad i = 1 \;\; to \;\; m \qquad (13)$$

det $_{i,m}$() stands for the determinant of a squared matrix box including the elements $\partial^2 R/\partial x_j \partial x_k$, with *j*, *k*= *i* to *m*. The box is taken from the *m*x*m* Hessian matrix. Eq (12) at the maximum yields:

$$\frac{\partial R}{\partial x_i} = 2\frac{4\pi}{\lambda}\left\{ -\,\mathrm{Im}\left[sN_i \cos\theta_i \bar{t}_i \right] + 2\,\mathrm{Im}\left[r_{Inc,1} sN_i \cos\theta_i \bar{t}_1 \bar{t}_i \right] - \left|1 - r_{Inc,1}{}^2\right|^2 \mathrm{Im}\left[N_i \cos\theta_i \bar{t}_1{}^* \bar{t}_i \right] \right\}$$

$$= 0, \qquad\qquad i = 1 \;\; to \;\; m, \qquad (14)$$

Im[ ] stands for the imaginary part of the expression in brackets.

2.1 First order approximation in ΔN_i

We start solving the first order expansion of Eq. (14):

$$\frac{\partial R}{\partial x_i} = -2\frac{4\pi}{\lambda}\operatorname{Im}\left[sN_i\cos\theta_i\bar{t}_i\right] = 0, \quad i = 1 \quad to \quad m \qquad (15)$$

We will call $\{x_{max,l}{}^{0}\}$ =($x_{max,1}{}^{0}$, $x_{max,2}{}^{0}$,...,$x_{max,m}{}^{0}$) the point at which the first order expansion of the first derivative of the reflectance has a maximum. Operating in a manner similar to that in Ref. 1 we convert Eqs. (15) into the following equations:

$$\operatorname{Im}\left[su_1\exp\beta_1{}^{0}\right] = 0 \qquad (16-1)$$

$$\operatorname{Im}\left[\frac{u_i}{u_{i-1}}\exp\beta_i{}^{0}\right] = 0, \quad i = 2 \quad to \quad m \qquad (16-2)$$

where the convenient complex numbers u_i are defined by:

$$u_i = N_i\cos\theta_i f_i, \quad i = 1 \quad to \quad m \qquad (17)$$

The superscript 0 in Eq. (16) and in the subsequent equations refers to the solution of the first order reflectance expansion in ΔN_i. Those thickness values are of zero order in ΔN_i. The optimum film thicknesses are obtained from the Eqs. (16):

$$tan\varphi_{max,1}{}^{0} = -\frac{\operatorname{Im}[su_1]}{\operatorname{Re}[su_1]}; \qquad (18-1)$$

$$tan\varphi_{max,i}{}^{0} = -\frac{\operatorname{Im}[u_i/u_{i-1}]}{\operatorname{Re}[u_i/u_{i-1}]}; \quad i = 2 \quad to \quad m \qquad (18-2)$$

where the optimum film thickness is defined by:

$$\varphi_{max,i}{}^{0} = \frac{4\pi n_i x_{max,i}{}^{0}}{\lambda}, \quad i = 1 \quad to \quad m \qquad (19)$$

From Eq, (18), φ_i is equal to the phase of the complex numbers su_1 (i=1) or u_i/u_{i-1} (i=2 to m), with opposite sign.

$\tan^{-1}$[-Im[su_1]/Re[su_1]] and $\tan^{-1}$[-Im[u_i/u_{i-1}]/Re[u_i/u_{i-1}] in Eqs. (18) are multi-valued functions: if φ_i^0 is a solution of any of the Eqs. (18), then any number differing from φ_i^0 by an integer number times π is also a solution. From the different solutions of Eqs. (18), those satisfying all the conditions (13) will be maxima and those not satisfying at least one of the conditions (13) will be either minima or saddle points. Since we are dealing with absorbing films, we will limit our interest to materials that provide a reflectance maximum with φ_i^0 values between 0 and π, that correspond to sub-quarterwave films.

We now determine the conditions for the refractive indices of the materials in order to provide an m-dimensional reflectance maximum with m sub-quarterwave films over an opaque substrate. The case where the sub-quarterwave films over an opaque substrate corresponds to a minimum or a saddle point will not be analyzed here. The conditions for a solution of Eqs. (18) to be a reflectance maximum with sub-quarterwave films are obtained from Eqs. (13) and can be expressed as:

$$\mathrm{Im}[su_1] > 0 \qquad (20-1)$$

$$\mathrm{Im}[u_i / u_{i-1}] < 0, \quad i = 2 \quad to \quad m \qquad (20-2)$$

where it was taken into account that $0<\varphi_i\leq\pi$, i=1 to m.

The conditions (20) can be expressed in a more convenient form by expanding the Fresnel coefficients to first order in ΔN_i. Different expressions are obtained for *s* (TE) and *p* (TM) polarized radiation:

$$s \quad polarization:$$

$$\mathrm{Im}\left[\frac{s\Delta N_1}{\cos\theta_1}\right] < 0 \qquad (21-1)$$

$$\mathrm{Im}\left[\frac{\Delta N_i}{\Delta N_{i-1}}\right] < 0, \quad i = 2 \quad to \quad m \qquad (21-2)$$

$$p \quad polarization:$$

$$\mathrm{Im}\left[s\Delta N_1 \cos\theta_1 \left(1 - \frac{N_0{}^2 sin^2\theta_0}{N_1{}^2 \cos^2\theta_1}\right)\right] < 0 \qquad (21-3)$$

$$\mathrm{Im}\left[\frac{\Delta N_i}{\Delta N_{i-1}}\right] < 0, \quad i = 2 \quad to \quad m \qquad (21-4)$$

When a set of m+1 materials with refractive indices N_i, i=1 to m+1, satisfy the Eqs. (21), a sub-quarterwave multilayer can be designed with a reflectance enhancement. The Eqs. (21) may not be satisfied for the materials arranged in a certain order, but the same refractive indices in a different order may satisfy it. Besides, a set of materials may not satisfy Eqs. (21), but a subset may satisfy it. Criteria for the selection of materials and their arrangement in the multilayer will be given in Section 2.2. Parameter *s* depends on the Fresnel coefficients at the interface with refractive indices N_{Inc} and N_1 and therefore, it depends on the polarization degree. The conditions for the outermost film (21-1) and (21-3) depend on the angle of incidence; however, the conditions for all the other films do not depend on the angle of incidence and are the same that in Ref. 1. At normal incidence obviously all conditions for *s* or *p* polarized radiation are equivalent. The conditions (21) are important in the current theory because they provide the selection rules for the materials so that the multilayer has an *m*-dimensional maximum for sub-quarterwave films. The current theory provides the method to find the correct combination of materials to achieve a reflectance enhancement. The first step to design the current sub-quarterwave multilayers is then to search for materials with the adequate refractive indices and arrange them in the correct order from the outermost to the innermost film and to the substrate so that they satisfy the conditions (21). In a second step we calculate approximate values for the film thickness using Eqs. (18). The third step is to calculate the reflectance at the maximum. This can be performed exactly through computer calculation once we obtained the optimum film thicknesses. In any event, approximate calculation of the reflectance at the maximum is obtained with the current theory to first order in ΔN_i:

$$R = R_{Inc,1} + 2|s|\sum_{j=1}^{m} \frac{|f_j|\operatorname{Re}\left[N_j \cos\theta_j\right]}{\left|N_j \cos\theta_j\right|} \exp\left(-\frac{4\pi}{\lambda}\sum_{k=1}^{j} \operatorname{Im}\left[N_k \cos\theta_k\right] x_k{}^0\right) + O\left(\Delta N^2\right) \quad (22)$$

and will be used in Section 2.2 to obtain an illustrative picture of the reflectance enhancement through sub-quarterwave multilayers. It will be also used in Section 3 to compare current theory with exact calculations.

All the above equations are valid for either *s* or *p* polarization, except the conditions (21) where both polarization states were explicitly shown. In the more general case that radiation is partially polarized, simple equations valid for any polarization are readily obtained from the above equations. The radiation polarization is described through the degree of polarization:

$$p = \frac{I_p - I_s}{I_p + I_s} \qquad (23)$$

The conditions for the multilayer to have a maximum for sub-quarterwave films are easily generalized from Eqs. (21) to include partial polarization:

$$\mathrm{Im}\left[\Delta N_1 \left\{\frac{1-p}{2}\frac{s^s}{\cos\theta_1} + \frac{1+p}{2} s^p \cos\theta_1 \left(1 - \frac{N_0{}^2 sin^2\theta_0}{N_1{}^2 \cos^2\theta_1}\right)\right\}\right] < 0 \qquad (24-1)$$

$$\mathrm{Im}\left[\frac{\Delta N_i}{\Delta N_{i-1}}\right] < 0, \;\; i = 2 \;\; to \;\; m \qquad (24-2)$$

Optimum film thickness is now given by:

$$tan\varphi_{max,1}{}^0 = -\frac{\mathrm{Im}\left[\frac{1-p}{2} s^s u_1{}^s + \frac{1+p}{2} s^p u_1{}^p\right]}{\mathrm{Re}\left[\frac{1-p}{2} s^s u_1{}^s + \frac{1+p}{2} s^p u_1{}^p\right]}; \qquad (25-1)$$

$$tan\varphi_{max,i}{}^0 = -\frac{\mathrm{Im}\left[\dfrac{\frac{1-p}{2} s^s u_i{}^s + \frac{1+p}{2} s^p u_i{}^p}{\frac{1-p}{2} s^s u_{i-1}{}^s + \frac{1+p}{2} s^p u_{i-1}{}^p}\right]}{\mathrm{Re}\left[\dfrac{\frac{1-p}{2} s^s u_i{}^s + \frac{1+p}{2} s^p u_i{}^p}{\frac{1-p}{2} s^s u_{i-1}{}^s + \frac{1+p}{2} s^p u_{i-1}{}^p}\right]}; \;\; i = 2 \;\; to \;\; m \qquad (25-i)$$

where $u_i{}^{s(p)}$ are defined in Eq. (17) using the *s* (*p*) Fresnel coefficients.

The reflectance at the maximum for partially polarized radiation is immediately obtained to first order in ΔN_i from Eq. (22) using:

$$R = \frac{1-p}{2} R_s + \frac{1+p}{2} R_p \qquad (26)$$

2.2 Reflectance dependence on the refractive indices of the materials

In this Section we will provide simple concepts to help the understanding of the theory developed in Section 2.1. We will show the way to select the materials for sub-quarterwave multilayers and the reflectance enhancement obtained with these multilayers. To do that, we simplify Eq. (22) by substituting the refractive index of every layer in the multilayer with an average value $N_c=n_c+\mathrm{i}k_c$:

$$R = R_{Inc,1} + \frac{|s| n_c}{n_c^{\,2} + k_c^{\,2}} \sum_{j=1}^{m} \left|\Delta N_j\right| \exp\left(-\frac{4\pi}{\lambda} k_c \sum_{k=1}^{j} x_k^{\,0} \right) \qquad (27)$$

We maintained the refractive index increments and assumed normal incidence for simplicity. The modifications are of second order in ΔN_i. Therefore, Eq. (27) is still correct to first order in ΔN_i. From Eq. (27), the multilayer reflectance enhancement over $R_{Inc,1}$ is an addition of one term per film, that, apart from a common factor, it is equal to the modulus of the refractive index increment ($|\Delta N_j|$ for the j-th layer) reduced by the exponential term of the transmittance through the accumulated thickness of the first to the j-th layers. The reflectance enhancement is then smaller when we go deeper into the multilayer. This is responsible for a reflectance enhancement saturation after a few films due to the high absorption properties of the materials. Eq. (18) gives the film thicknesses from parameters φ_i. As mentioned in Section 2.1, φ_i are the phase of the complex numbers su_1 (for φ_1) and u_i/u_{i-1} (for φ_i, i=2 to m) with opposite sign. These complex numbers can be rewritten in the above assumptions:

$$su_1 = -\frac{s\Delta N_1}{2} \qquad (28-1)$$

$$\frac{u_i}{u_{i-1}} = \frac{\Delta N_i}{\Delta N_{i-i}}, \quad i = 2 \quad to \quad m \qquad (28-2)$$

Eq. (28-1) depends on the complex parameter *s*. Eq. (28-2) only depends on the refractive index increments. The phase of φ_i, $i \geq 2$, is the angle between the refractive index increments when we plot them as vectors with a common origin, obtained by rotation from ΔN_i to ΔN_{i-1}. The *i*-th film thickness is proportional to φ_i through Eq. (19). Eq. (27) can be rewritten as:

$$R = R_{Inc,1} + \frac{|s| n_c}{n_c^{\ 2} + k_c^{\ 2}} \sum_{j=1}^{m} |\Delta N_j| \exp\left(-\frac{k_c}{n_c} \overline{\varphi}_j \right), \quad with \quad \overline{\varphi}_j = \sum_{k=1}^{j} \varphi_k \qquad (29)$$

The condition (20-2) for a reflectance enhancement through sub-quarterwave multilayers of films 2 to *m* results in a restriction on the rotation angle from ΔN_i to ΔN_{i-1} between 0 and π. In order to have an illustrative picture of the above we plot in Fig. 1 a simple example where we assume that the refractive indices of all the materials in the multilayer are within a circumference of radius *l* centered at N_c. *l* is assumed to be small so that all the refractive indices are close enough as required by the theory developed in Section 2.1. Every ΔN_i is represented by a vector starting and terminating at two different points in the circumference.

The material for the outermost film N_1 and the refractive index increment ΔN_1 have to be selected so that the condition (20-1) is satisfied. This is done by any ΔN_1 whose phase added to that of parameter *s* is between 0 and π. The parameter *s* depends on the outermost material refractive index. Since the refractive indices of the materials are within a circumference, the condition (20-1) will be necessarily satisfied with some materials, and not with others. One of the former materials will be selected as the outermost material.

Once we select a proper material for the outermost film and a proper ΔN_1, the condition (20-2) is satisfied by selecting the second to m-layer materials and the substrate with refractive indices given by clockwise rotation in the circumference of less than π radians per film. With counterclockwise rotation we would get sub-quarterwave multilayers giving a reflectance minimum.

When going deep into the multilayer and after using all the available refractive indices we can start again with N_1, completing a full period, and we can continue adding more periods underneath as long as the reflectance enhancement is worth adding more layers to the multilayer. In this sense the current multilayers are similar to the standard multilayers with two materials; in the standard case one full period is completed after two layers, and many periods are completed when the refractive index oscillates between the two extreme values. The difference is that with sub-quarterwave multilayers the refractive index takes three or more values per period instead of two. The integrated optical path through one full period is $\lambda/2$ both for sub-quarterwave multilayers and for standard multilayers.

From the Eq. (29), the reflectance enhancement over $R_{Inc,1}$ is proportional to the circumference radius l, which is demonstrated by expressing $|\Delta N_j|=2l\sin(\alpha_i/2)$, where α_i is the angle subtended by ΔN_j from the circumference center N_c. In other words: the larger the difference in the refractive indices, the higher the reflectance increase. The film thickness can be expressed in terms of the α_i angles through:

$$\varphi_i = \frac{1}{2}\left(\alpha_{i-1} + \alpha_i\right) \qquad (30)$$

In the more particular case were all φ_i are the same, it is $\varphi = \alpha$. In this case the increase in reflectance due to the *i*-th film is determined by the angle subtended by the *i-th* refractive index increment from the circumference center N_c, and by the accumulated angle starting with the first film. The accumulated angle can be larger than 2π for a multilayer covering more than one period. Another important parameter is k_c/n_c. The lower the parameter, the larger the reflectance increase per added film and the more layers provide a significant reflectance enhancement.

We now investigate the effect of adding or eliminating a layer in the multilayer. Let R be the multilayer reflectance given by Eq. (29) and let R' be the reflectance for the same multilayer except that we eliminate the *i*-th layer, maintaining the rest of the layers. Let this multilayer be optimized; therefore, the reflectance has an *m*-1-dimensional maximum. In the new multilayer the refractive index difference at the *i*-th film is now $|\Delta N_i + \Delta N_{i+1}|$ and the *i*-th film thickness is given by the addition of the angles associated to ΔN_i and ΔN_{i+1}. Reflectance difference for the two multilayers is given by:

$$R - R' = \frac{|s| n_c}{{n_c}^2 + {k_c}^2}$$
$$\times \left[|\Delta N_{i-1}| \exp\left(-\frac{k_c}{n_c}\overline{\varphi}_{i-1}\right) + |\Delta N_i| \exp\left(-\frac{k_c}{n_c}\overline{\varphi}_i\right) - |\Delta N_{i-1} + \Delta N_i| \exp\left(-\frac{k_c}{n_c}\overline{\varphi}_i\right)\right] \quad (31)$$

that straightforwardly yields:

$$R - R' > \frac{|s| n_c}{n_c^2 + k_c^2} \left[|\Delta N_{i-1}| + |\Delta N_i| - |\Delta N_{i-1} + \Delta N_i| \right] \exp\left(-\frac{k_c}{n_c} \overline{\varphi}_i \right) > 0 \qquad (32)$$

From Eq. (32), the reflectance of the sub-quarterwave multilayer increases when inserting in the multilayer one film of a new material satisfying condition (20-2). The reflectance enhancement of the multilayer with one more material is due to two facts. One is that the accumulated length of the refractive index increments is higher when more materials are added. The other is that when we have a new material inserted between two other materials, the accumulated thickness and hence the absorption of the multilayer up to the new material is lower than for the next material underneath. In other words: with the sub-quarterwave films we make an optimum use of the incoming energy by reflecting it at interfaces as close as possible to the outer surface. The highest possible reflectance would be obtained by using every refractive index in the circumference, resulting in an infinity of films of infinitesimal thickness given by $dx=\lambda d\varphi/4\pi n_c$, where $d\varphi$ is the infinitesimal angle subtended by the refractive index increment corresponding to the material between $N(\varphi)$ and $N(\varphi+d\varphi)$.

2.3 Second order approximation in ΔN_i

In this Section a more accurate multilayer design will be obtained by expanding the multilayer reflectance to the second order in ΔN_i as given in Eq. (11). We will use the thickness values $\{x_{max,l}^{\;0}\}$ derived in Section 2.1 as the initial values of the calculation and will obtain thickness corrections.

We will assume that the maximum of the reflectance is at a certain $\{x_{max,l}{}^{0} + \Delta x_{max,l}\}$ and we want to obtain the corrections $\{\Delta x_{max,l}\}$. The second order Taylor series expansion of the reflectance with respect to the small corrections $\{\Delta x_{max,l}\}$ is:

$$R\left(\left\{x_{max,l}{}^{0} + \Delta x_{max,l}\right\}\right) = R\left(\left\{x_{max,l}{}^{0}\right\}\right) + \sum_{j=1}^{m} \frac{\partial R}{\partial x_j}\left(\left\{x_{max,l}{}^{0}\right\}\right)\Delta x_j$$
$$+ \frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m} \frac{\partial^2 R}{\partial x_j \partial x_k}\left(\left\{x_{max,l}{}^{0}\right\}\right)\Delta x_j \Delta x_k + O\left(\Delta x^3\right) \quad (33)$$

As a reminder $\{x_{max,l}{}^{0}\}$ are the thicknesses maximizing the first order expansion of the reflectance, but not the exact reflectance. The reflectance gradient at the exact *m*-dimensional maximum is zero:

$$0 = \frac{\partial R}{\partial x_i}\left(\left\{x_{max,l}{}^{0} + \Delta x_{max,l}\right\}\right)$$
$$= \frac{\partial R}{\partial x_i}\left(\left\{x_{max,l}{}^{0}\right\}\right) + \sum_{j=1}^{m} \frac{\partial^2 R}{\partial x_i \partial x_j}\left(\left\{x_{max,l}{}^{0}\right\}\right)\Delta x_j + O\left(\Delta x^2\right), \quad i = 1 \ \ to \ \ m \quad (34)$$

Rearranging Eq. (34):

$$\sum_{j=1}^{m} \frac{\partial^2 R}{\partial x_i \partial x_j}\left(\left\{x_{max,l}{}^{0}\right\}\right)\Delta x_j = -\frac{\partial R}{\partial x_i}\left(\left\{x_{max,l}{}^{0}\right\}\right) + O\left(\Delta x^2\right), \quad i = 1 \ \ to \ \ m \qquad (35)$$

Eq. (35) is correct to first order in Δx_k. By using the second order reflectance expansion in ΔN_i in Eq. (35) we can calculate a correction $\{\Delta x_{max,l}\}$. The first derivative of the reflectance was shown in Eq. (12). The second derivative is calculated from Eq. (14):

$$\frac{\partial^2 R}{\partial x_i \partial x_j}\left(\left\{x_{max,l}{}^0\right\}\right)$$
$$= 2\left(\frac{4\pi}{\lambda}\right)^2 \left\{ -\operatorname{Re}\left[sN_i \cos\theta_i N_j \cos\theta_j \bar{t}_i{}^0\right] + 2\operatorname{Re}\left[r_{Inc,1} sN_i \cos\theta_i N_j \cos\theta_j \left(\bar{t}_j{}^0 + \bar{t}_1{}^0\right)\bar{t}_i{}^0\right]\right.$$
$$\left. - \left|1 - r_{Inc,1}{}^2\right|^2 \operatorname{Re}\left[N_i \cos\theta_i \bar{t}_i \left(N_j \cos\theta_j \bar{t}_1{}^{0*} - N_j{}^* \cos\theta_j{}^* \bar{t}_j{}^{0*}\right)\right]\right\}, \quad i = 1 \;\; to \;\; m, \quad j \le i \quad (36)$$

The second derivative is symmetric by interchanging i and j. $\{x_{max,l}{}^0\}$ is of zero order in ΔN_i. In Eq. (35) the second order expansion of the reflectance gradient at $\{x_{max,l}{}^0\}$ is a second order term in ΔN_i. Since the second derivative of the reflectance is of first order in ΔN_i, the thickness corrections $\{\Delta x_{max,l}\}$ are of first order in ΔN_i.

Eq. (35) is then a set of m linear algebraic equations with m unknowns Δx_k, k=1 to m, and can be easily solved once we calculate all the coefficients given by the first and the second derivatives of the reflectance. The reflectance at the new maximum is obtained from Eqs. (33) and (35):

$$R\left(\left\{x_{max,l}{}^0 + \Delta x_{max,l}\right\}\right) = R\left(\left\{x_{max,l}{}^0\right\}\right) - \frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m} \frac{\partial^2 R}{\partial x_j \partial x_k}\left(\left\{x_{max,l}{}^0\right\}\right)\Delta x_j \Delta x_k + O\left(\Delta x^3\right) \quad (37)$$

For the general case of any polarization given by the parameter p Eqs. (35) to (37) are generalized using Eq. (26) and the following substitutions:

$$\frac{\partial R}{\partial x_i}=\frac{1-p}{2}\frac{\partial R_s}{\partial x_i}+\frac{1+p}{2}\frac{\partial R_p}{\partial x_i}$$
$$\frac{\partial^2 R}{\partial x_i \partial x_j}=\frac{1-p}{2}\frac{\partial^2 R_s}{\partial x_i \partial x_j}+\frac{1+p}{2}\frac{\partial^2 R_p}{\partial x_i \partial x_j} \quad (38)$$

An interesting feature of Eq. (35) is that it can be employed iteratively. After obtaining $\{\Delta x_{max,l}\}$ we can redefine $\{x_{max,l}{}^1\}=\{x_{max,l}{}^0+\Delta x_{max,l}\}$ and recalculate the first and second derivatives at the new initial values and obtain new thickness increments.

3. Examples

In this Section we provide examples of applications of sub-quarterwave multilayers in cases where the available single layer coatings give a moderate reflectance in the EUV. One example given in Ref. 1 is reanalyzed in Section 3.1 and new accurate values are obtained for the film thickness and the multilayer reflectance with the improved theory. An example of a multilayer optimized at non-normal incidence is shown in Section 3.2.

3.1 Multilayers with high reflectance at 83.4 nm at normal incidence

We address here the design of a multilayer with the highest possible reflectance at the OII 83.4 nm spectral line, which is an important spectral line for atmospheric physics. At this wavelength, the available single layer coatings, such as SiC films, have moderate reflectance, and a reflectance enhancement is desired. The multilayer design is optimized at normal incidence and assumes vacuum as the incidence medium. The refractive index for ion-beam-deposited (IBD) SiC[2] (0.58,1.07), IBD B_4C[3]

(0.816,1.246), IBD C[4] (1.16,1.29), Al_2O_3[5] (1.350,1.147) and MgF_2[6] (1.59,0.49) will be used in the calculations. All the above materials have useful refractive indices for the application of the current theory. The above refractive indices are not close to each other, as required for the application of the current theory: the real and the imaginary parts of the refractive indices change by a factor of three and over two, respectively. Nevertheless, the current theory will provide us with accurate values for the film thickness and reflectance in this example, which proves that the theory may be also applied for materials with large refractive index differences (the most usual case).

Conditions (20) and the equivalent (21) are satisfied at 83.4 nm at normal incidence with sub-quarterwave films for any selection of materials starting with SiC as the outermost layer and going deeper into the multilayer in the order given by B_4C/ C/ Al_2O_3/ MgF_2/SiC. Any such selection results in a multilayer with a reflectance enhancement. The refractive index of these materials is also plotted in Fig. 1. In the figure it can be observed that the materials are arranged in a way that reminds of a circumference, as in the example described in Section 2.2. The multilayers were calculated using the current improved theory in two steps: we first obtained the film thickness to first order in ΔN_i using Eqs. (18) and then we calculated the film thickness corrections through Eqs. (35) and (36). This was performed for the following selection of multilayers using the above materials: SiC/ C, SiC/ MgF_2, SiC/ Al_2O_3/ MgF_2, SiC/ B_4C/ C/ MgF_2, SiC/ B_4C/ C/ Al_2O_3/ MgF_2 and SiC/ B_4C/ C/ Al_2O_3/ MgF_2/SiC. The last multilayer has a SiC layer both for the outermost and for the innermost multilayer, which provides a further reflectance increase; this combination was not analyzed in Ref.1. The exact film thickness of the optimum multilayers was also calculated by a search of the film thickness by trial and error. The film thickness calculated with the theory of Ref. 1, the ones obtained with the current theory to first and second order and the exact value at which the multilayer has the *m*-dimensional maximum are compared in Table 1. The thickness accuracy is better for the current first order theory than for the theory of Ref. 1, particularly for the outermost films. The thicknesses calculated up to the second order provide a better approximation to the exact values. The important thickness deviation at some of the deep layers results in a negligible effect on the reflectance due to the low contribution of the deep interfaces. The exact reflectance of the multilayers designed with the different approaches is also shown in Table 1. The first, and even more the second order solution provide multilayers with a reflectance remarkably close to the exact optimum reflectance. The multilayer reflectance maximum was also calculated with Eqs. (22) and (37) for the first and for the second order multilayer designs, respectively, given in Table 1. The results are given in Table 2, which compares the reflectance at the maximum obtained with the current theory for the first and second order expansions, to that obtained by exact calculation. For the SiC/ B_4C/ C/ Al_2O_3/ MgF_2/SiC multilayer, the second order film thickness calculation was performed by three iterations as described in Section 2.3, whereas for all the other multilayers only one iteration was necessary for convergence. The reflectance calculated with the current theory up to the first and second order reproduces the exact values with remarkable accuracy. The calculated reflectance is more accurate with the second than with the first order theory. An optimized multilayer with a further B_4C layer under the SiC film would provide a positive but negligible reflectance enhancement. The same is obtained by adding more layers of the same materials under the innermost layer. The reason for this is that the radiation reaching those inner layers is negligible.

3.2 Multilayers with high reflectance at 53.6 nm at 45° incidence

In the current section we design sub-quarterwave multilayers optimized at an angle away from the normal incidence for two interesting polarization states: *s* polarized and non-polarized radiation. We address the design of a multilayer with the highest possible reflectance at the HeI, 53.6 nm spectral line, which is an important spectral line for astrophysics. The design is made at 45° incidence angle and assumes vacuum as the incidence medium. The refractive index for IBD B_4C[3] (0.517,0.575), IBD Mo[2] (0.55,0.73), Ir[7] (0.79,0.96) and C_{60}[8] (0.804,0.477) will be used in the calculations. All the above materials have useful refractive indices for the application of the current theory.

Conditions (20) and the equivalent (21) are satisfied at 53.6 nm at 45° incidence and any polarization with sub-quarterwave films for any selection of materials in the order B_4C/ Mo/ Ir/ C_{60}, starting with the outermost layer. Every selection gives rise to sub-quarterwave multilayers with an *m*-dimensional reflectance maximum, *m* being the number of layers over the opaque substrate. The refractive index of these materials is also plotted in Fig. 1. In this example it can be observed again that the materials are arranged in a manner that reminds of a circumference. Table 3 compares the reflectance for a selection of the possible multilayers with the above materials to that of single layers. The film thickness calculated with the current theory to first and second order and the exact value at which the multilayer has the *m*-dimensional maximum are compared in Table 3. The exact reflectance of the multilayer designed with the different approaches is also shown in Table 3. Similar to the example in Section 3.1, the thickness accuracy is better for the second than for the first order values. The first, and even more the second order solution provide multilayers with a reflectance remarkably close to the exact optimum reflectance for the two polarization states analyzed. Again the thickness deviation from the exact values has a small effect on the multilayer reflectance.

The reflectance as a function of wavelength at 45° incidence for *s*-polarized radiation is shown in Fig. 2 for a simple multilayer of B_4C on Ir, along with the reflectance of single layers of Ir and B_4C. The multilayer was optimized for the highest possible reflectance at 53.6 nm, 45° incidence and *s*-polarized radiation. A reflectance enhancement compared to single layers is obtained at 53.6 nm. This enhancement extends over a wide spectral region between 66 and 40 nm, and probably below. The broadband reflectance enhancement is a characteristic of sub-quarterwave multilayers with highly absorbing films, as it was shown in Ref. 1 for multilayers optimized at 83.4 nm. Remarkably, a reflectance of 0.39 is calculated at 40 nm. The reflectance enhancement is higher at short wavelengths than at the target (53.6 nm) because the absorption of the materials decreases with wavelength and more layers can provide a significant reflectance enhancement.

The reflectance versus the angle of incidence at 53.6 nm for *s*-polarized radiation is shown in Fig. 3 for a multilayer of B_4C/ Mo/ Ir/ C_{60} optimized for highest reflectance at 53.6 nm, 45° incidence and *s*-polarized radiation. The reflectance for single layers of B_4C, Mo, and Ir are also shown for comparison. Mo is the single layer with a higher reflectance. Since Mo oxidizes over time under normal atmosphere with an important reflectance decrease, comparison of multilayer reflectance to that of single B_4C films is more informative. The reflectance enhancement of the multilayer compared to the single layers is obtained over a wide range of incidence angles. Even though the multilayer was optimized at 45°, the highest reflectance enhancement is obtained at normal incidence. The reasons for this are the increase of the radiation absorption of the materials from normal to grazing incidence and the high tolerance of sub-quarterwave multilayers of highly absorbing materials.

In the examples shown in Section 3, the compatibility between the materials, and the multilayer stability over time have not been discussed. The theory presented assumes smooth and abrupt interfaces between adjacent layers of different materials, with no interdiffusion through the interface. In practice, the real multilayers might be unstable. The proposed multilayers must be understood as examples of the method to combine materials for sub-quarterwave multilayers and the achievable reflectance enhancement. The real reflectance enhancements and stability over time would need to be experimentally determined for any set of materials.

Conclusions

A theory of sub-quarterwave multilayers with highly absorbing materials has been developed. The theory enables the design of sub-quarterwave multilayer coatings with reflectance enhancement over that of single layers, which are particularly useful for the FUV-EUV over the spectral range ~50-125 nm, where transparent materials, necessary to prepare standard multilayers, are not available. The multilayers may be optimized for the highest possible reflectance at any desired incidence angle and radiation polarization. The theory provides an accurate approach to the design of a new concept of multilayer coatings with more than two materials that goes beyond the standard limit of two materials.

The theory provides the selection rules for the multilayer construction, along with simple equations to calculate the layer thicknesses and the multilayer reflectance. The layer thicknesses are calculated using the expansion of the multilayer reflectance to the second order in the refractive index differences between adjacent films. The multilayer film thicknesses are obtained in two steps. In a first step the zero order film thicknesses are calculated. In a second step the above values are used as the initial values to obtain first order thickness corrections. The sub-quarterwave multilayers optimized at a given wavelength and incidence angle provide reflectance enhancements that extend over a wide spectral range and over a wide range of incidence angles. The reflectance enhancement over that of single layers is highest at normal incidence and decreases towards grazing incidence.

The reflectance enhancement with sub-quarterwave multilayers increases with the refractive index differences, and with the number of different materials used, as long as they satisfy the selection rules. The material selection rules are satisfied when the refractive index differences between adjacent layers plotted as vectors in the complex plane rotate clockwise from the outermost to the innermost layer in steps of less than π radians per film. Once all the available refractive indices are used, one per film in the multilayer, further reflectance enhancement is obtained by starting again with the first refractive index, hence completing a full period. More periods can be added underneath as long as the reflectance enhancement is worth adding more layers.

The multilayer reflectance enhancement is an addition of one term per film, which, apart from a common factor, it is equal to the modulus of the refractive index increment reduced by the exponential term of the transmittance through the accumulated thickness of all the outermost layers up to that layer. The deeper we go in the multilayer, the smaller the reflectance enhancement, which results in a reflectance enhancement saturation after a few films due to the high absorption properties of the materials. By taking an average refractive index among the materials used $N_c=(n_c, k_c)$, the lower the parameter k_c/n_c, the larger the reflectance increase per added film and the more layers provide a significant reflectance enhancement.

New research is underway to apply the new theory of sub-quarterwave multilayers to provide further reflectance enhancements at wavelengths below 50 nm, where standard multilayers with many layers are nowadays available.

Acknowledgments

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Table 1. Film thickness of different sub-quarterwave multilayers calculated by four different methods: the one from Ref. 1, the current approach to first order in ΔN_i, the current approach to second order in ΔN_i, and by exact calculation. The exact reflectance for the four designs is also shown. The optimization was performed for the highest possible reflectance at 83.4 nm at normal incidence. The reflectance of single layers of the materials is also shown.

Calculation method	Thickness (nm)						Exact reflectance
	SiC	B_4C	C	Al_2O_3	MgF_2	SiC	
Exact	opaque						0.3629
Exact		opaque					0.3271
Exact			opaque				0.2669
Exact				opaque			0.2103
Exact					opaque		0.0847
Ref. 1	12.63		opaque				0.3902
Current; first order	11.04		opaque				0.3916
Current; second order	10.70		opaque				0.3916
Exact	10.72		opaque				0.3916
Ref. 1	22.74				opaque		0.3778
Current; first order	17.58				opaque		0.3832
Current; second order	17.34				opaque		0.3832
Exact	17.36				opaque		0.3832
Ref. 1	15.63			6.49	opaque		0.3942
Current; first order	13.12			6.61	opaque		0.3994
Current; second order	11.37			7.15	opaque		0.4003
Exact	11.71			6.54	opaque		0.4004
Ref. 1	9.44	4.18	6.89		opaque		0.4051
Current; first order	8.92	3.88	5.86		opaque		0.4074
Current; second order	7.23	3.40	6.74		opaque		0.4087
Exact	7.64	3.39	6.31		opaque		0.4088
Ref. 1	9.44	4.18	4.42	2.83	opaque		0.4065
Current; first order	8.92	3.88	4.50	2.20	opaque		0.4084
Current; second order	7.11	3.26	3.36	4.05	opaque		0.4101
Exact	7.58	3.32	3.70	3.26	opaque		0.4104
Ref.1	9.44	4.18	4.42	2.83	10.19	opaque	0.4149
Current; first order	8.92	3.88	4.50	2.20	12.37	opaque	0.4188
Current; second order	6.98	3.01	2.98	2.96	13.74	opaque	0.4262
Exact	7.06	2.83	2.98	2.60	13.65	opaque	0.4265

Table 2. The reflectance at 83.4 nm at normal incidence of different sub-quarterwave multilayers calculated by three different methods: the current theory to first order in ΔN_i (Eq. 22), the current theory to second order in ΔN_i (Eq. 37), and by the exact calculation. The film thicknesses were obtained by the optimization using the corresponding theory and are given in Table 1.

Multilayer	Reflectance approximations with different theories		
	1st order	2nd order	Exact
SiC/ C	0.3903	0.3904	0.3916
SiC/ MgF_2	0.3825	0.3826	0.3832
SiC/ Al_2O_3/ MgF_2	0.3971	0.3991	0.4004
SiC/ B_4C/C/ MgF_2	0.4046	0.4075	0.4088
SiC/ B_4C/C/ Al_2O_3/ MgF_2	0.4054	0.4099	0.4104

Table 3. Reflectance of sub-quarterwave multilayers optimized at 53.6 nm, 45° incidence angle and *s* polarized (p=-1) and non-polarized (p=0) radiation. The reflectance of single layers of the materials is also shown.

Calculation method	Thickness (nm)				polarization	Exact reflectance
	IBD B_4C	IBD Mo	Ir	C_{60}		
Exact	opaque				-1	0.4457
Exact		opaque			-1	0.4537
Exact			opaque		-1	0.3963
First order	7.75		opaque		-1	0.4791
Second order	7.41		opaque		-1	0.4792
Exact	7.44		opaque		-1	0.4792
First order	4.47	4.47	opaque		-1	0.4848
Second order	3.96	4.55	opaque		-1	0.4849
Exact	4.03	4.51	opaque		-1	0.4849
First order	4.47	4.47	14.72	opaque	-1	0.4855
Second order	3.95	4.50	14.75	opaque	-1	0.4857
Exact	4.01	4.48	14.66	opaque	-1	0.4857
Exact	opaque				0	0.3222
Exact		opaque			0	0.3297
Exact			opaque		0	0.2766
First order	9.01		opaque		0	0.3452
Second order	8.05		opaque		0	0.3455
Exact	8.13		opaque		0	0.3455
First order	5.17	5.25	opaque		0	0.3520
Second order	3.43	5.56	opaque		0	0.3529
Exact	3.68	5.61	opaque		0	0.3530
First order	5.17	5.25	14.20	opaque	0	0.3525
Second order	3.38	5.49	14.28	opaque	0	0.3535
Exact	3.65	5.58	14.18	opaque	0	0.3536

Figure Captions

Fig. 1. Representation in the complex plane of the refractive indices of the materials used in the examples of Section 3. A dot marks the outermost material used in the multilayer. The arrows point the next material in the film underneath. A circumference is also plotted to illustrate the case described in Section 2.2. The first refractive index increment and angle subtended from N_c are plotted. In the example, five materials with refractive indices N_1 to N_5 located in the circumference are assumed.

Fig. 2. Calculated reflectance versus the wavelength for single opaque layers of B_4C and Ir, and for the following multilayer that was optimized at 53.6 nm, 45° incidence and *s* polarization: 7.44 nm B_4C/ opaque Ir.

Fig. 3. Calculated reflectance versus the angle of incidence with respect to the normal for single opaque layers of Mo, B_4C, and Ir, and for the following multilayer that was optimized at 53.6 nm, 45° incidence and *s* polarization (starting with the outermost layer): 4.01 nm B_4C/ 4.48 nm Mo/ 14.66 nm Ir/ opaque C_{60}.

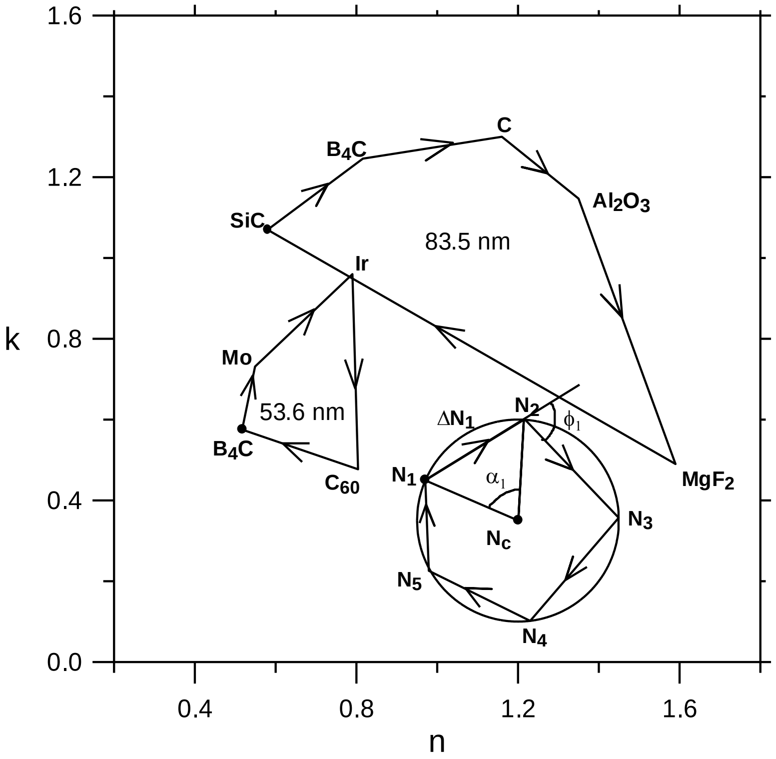

Fig. 1
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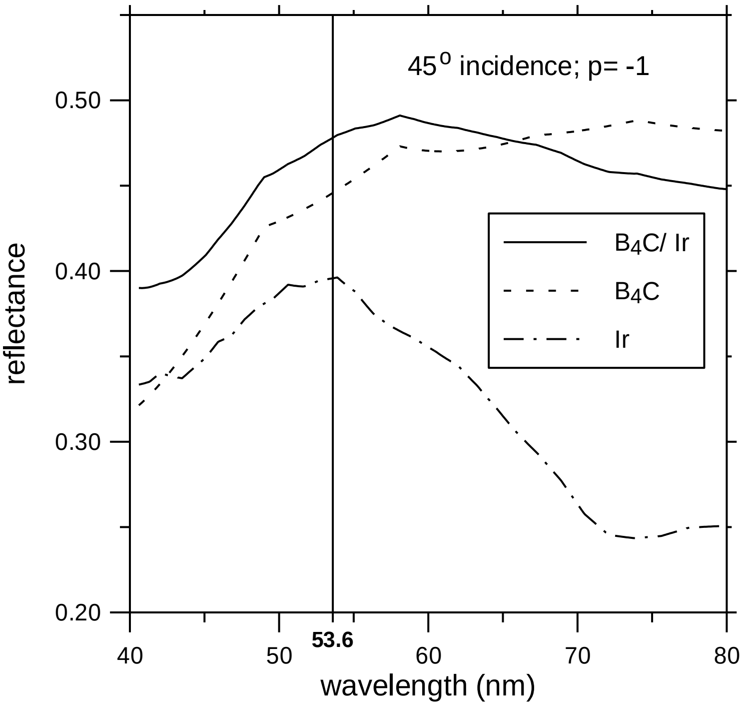

Fig. 2
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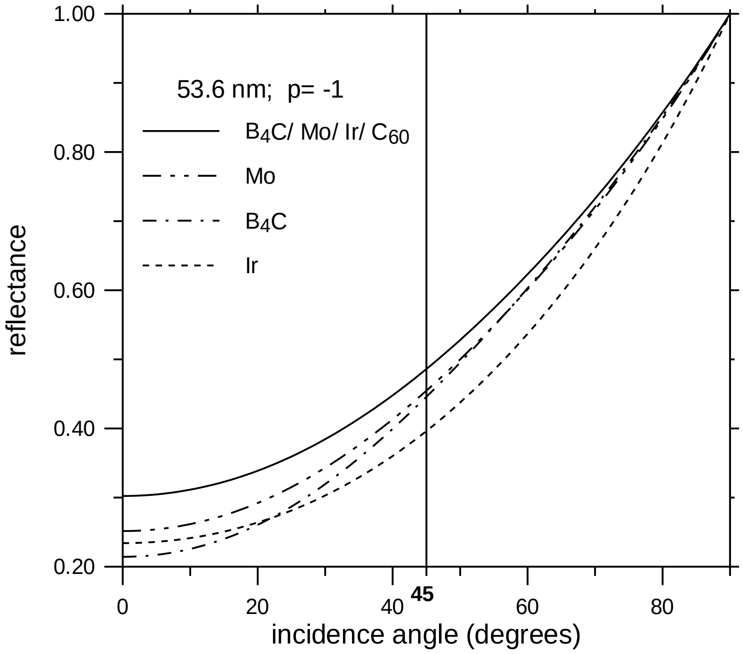

Fig. 3
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